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Increased Tail Dependence in Global Public Real Estate Markets

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This study examines the tail dependence of returns in international public real estate markets. By using the daily returns of real estate securities in seven cities/countries from 2000 to 2018, we analyze how the interdependence of international securitized real estate markets has changed since the Global Financial Crisis. We divide our sampling period into the pre-crisis, crisis, and post-crisis periods, and estimate both upper and lower tail dependence coefficients for each sub-period. Our empirical results confirm that most city/country pairs have changed from tail-independent to tail-dependent since 2007. Strong tail dependence persists during the crisis and post-crisis periods. The findings from the post-crisis sub-sample provide new evidence on increased tail dependence in the global real estate market in recent years. We conclude that international real estate securities still offer diversification benefits nowadays but to a lesser extent than in the pre-crisis period. Investing in the global real estate securities markets is beneficial for cross-region, mixed-asset portfolios.

Keywords

Tail Dependence, Real Estate Investment Trust, Mixed-Assets, Real Estate, Copula

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1. Introduction

Global public real estate markets have expanded remarkably in the last two decades. For example, the market capitalization of all real estate investment trusts (REITs) in the United States (US) increased from USD 138 billion in 2000 to over USD 907 billion in 2014¹. As of October 30, 2015, the market capitalization of all real estate securities globally is as high as USD 37 trillion². Meanwhile, diversifying into international stock markets offers limited benefits, as global stock markets are increasingly becoming interdependent. For example, Koch and Koch (1991) find an increase in correlation among daily stock returns in eight countries. Longin and Solnik (1995) examine the correlation of stock returns between the US and six other countries. They find a significant increase in correlation for four out of six pairs, especially in periods of high volatility. Against this backdrop, securitized real estate has gained popularity among investors given their relatively low correlation with other asset classes. International real estate diversification is found to be more effective than international stock and bond portfolios (e.g., Eichholtz, 1996; Hartzell et al., 1986; Liow et al., 2009). This conclusion holds for both mixed-asset and real-estate-only portfolios (see review in Worzala and Sirmans, 2003)³.

However, as world economies have become more integrated, the benefits of diversifying into international real estate markets have been diminishing as well. This trend seems to be more eminent during and after the Global Financial Crisis (e.g., Liow et al., 2015, Zimmer, 2015). Are international real estate markets still offering diversification benefits? If yes, where should investors put their money and how?

To answer these questions, it is essential to accurately measure the interdependence of return and volatility. The Pearson's linear correlation used to be the most popular measurement in this line of research. It also lies in the heart of the capital asset pricing model and the arbitrage pricing theory. However, this approach has received much criticism from both academic researchers and practitioners (e.g., Longin & Solnik, 2001; Rachev et al., 2005) because it assumes that the return series follows a multivariate normal distribution. On the contrary, financial data in the real world normally exhibit leptokurtic, skewness, and fat tails (Fama, 1965). As a result, the correlation method usually underestimates the risk of a portfolio and misleads investors into making suboptimal portfolio management decisions. Therefore, this

¹ Source: <https://www.reit.com/data-research/data/us-reit-industry-equity-market-cap>.

² Source:

<http://www.ftse.com/Analytics/FactSheets/Home/DownloadSingleIssue?openfile=open&issueName=ENHG>

³ Similar findings can be found in Eichholtz (1996), Okunev and Wilson (1997), Ling and Naranjo (1999), Mei and Hu (2000), Clayton and Mackinnon (2003), Wilson and Zurbrugg (2004), Liow and Yang (2005), Michayluk et al. (2006) and Cotter and Stevenson (2006).

approach should not be used to capture the dependence among financial time series (see evidence in Dowd, 2005; Dulguerov, 2009; and Zhou and Gao, 2012).

To address this issue, researchers have adopted a wide array of alternative methods to measure dependence, including Kendall's τ , Spearman's rank correlation, Blomqvist's β , the Gini coefficient, and copula-based methods (Cherubini et al., 2004). Recently, a growing number of studies have highlighted the benefits of using copulas to model dependence structures between financial series. The copula approach, proposed by Sklar (1959), was first introduced in the financial context by Embrechts et al. (2002). It is a flexible function that links univariate marginal distributions to form a joint distribution of these variables (Dowd, 2005) without imposing a multinormal distribution assumption onto the underlying variables. Copula models have been widely used in financial studies, including valuating financial derivatives, pricing portfolios and market risk management, and calculating dependence and value-at-risk, because of their simplicity and flexibility (e.g., Chen and Glasserman, 2008; Liu, 2015; Wang and Dyer, 2012; Weiß and Supper, 2013). The method has been found to be particularly beneficial in capturing tail dependence among time series (Aghakouchak et al., 2010, Chen et al., 2013 and Siburg et al., 2015).

Tail dependence is a measurement of the probability of the joint movement of two or more time series under extreme market conditions (e.g., boom or bust). It can describe the chances of observing the extreme value of an asset (market) given that another asset (market) shows an extreme value during market downturns and upturns. The analysis is more relevant and useful for understanding the co-movement between two or more real estate markets during stressful times (Muns and Bijlsma, 2015). Considering that the focus of our study is to investigate whether and how the interdependence of international real estate markets has changed since the Global Financial Crisis, we adopt the copula method to model the underlying distribution of returns accurately.

The copula method has been used in the real estate literature with promising results. For example, significant tail dependence has been identified in regional and international securitized real estate markets (Knight et al., 2005; Zhou and Gao 2012; Hoesli and Reka, 2013). Evidence shows that copula estimates are superior to estimates of the CCC-GARCH and DCC-GARCH models (Zhou and Gao, 2012), and that the distribution of securitized real estate returns is neither normal nor symmetric (Hoesli and Reka, 2013). Following this line of practice, we adopt the copula method in our investigation of tail dependence in international public real estate securities markets.

The closest existing studies to our work is Zhou and Gao (2012) and Hoesli and Reka (2013), where tail dependence in real estate securitized markets is analyzed by using dynamic Copula estimator. However, Zhou and Gao (2012) use data from 2000 to 2009, and Hoesli and Reka (2013) use data from the US, UK and Australia for the period of 1990–2010 only. Neither have sufficient

data to analyze tail dependence in real estate returns after the Global Financial Crisis. Yet, findings from this period (i.e., from 2010 onwards) are the most relevant for investors. To bridge this gap in the literature, we extend their work by including data from January 2000 to February 2018, which provide new evidence on tail dependence in the post-crisis period. We divide the whole sampling period into the pre-crisis, crisis, and post-crisis sub-periods, for which the coefficients of the upper and lower tail dependence are estimated to illustrate the effect of a financial crisis. The analysis is carried out by using data from seven cities/countries in the American, Asia-Pacific, and European regional markets. These countries/cities include the US, Hong Kong (HK), Japan, Australia, Singapore, the United Kingdom (UK), and France.

The main finding is that tail dependence in the international public real estate securities market has increased notably since the Global Financial Crisis. Almost all tail-independent city/country pairs have changed to tail-dependence. This pattern is consistent for the interdependence between stock market and both domestic and international real estate securities markets. Real-estate-only portfolios are more affected than mixed-asset portfolios. Although our analysis shows that diversifying into international real estate markets is still beneficial, the gains of such an approach have significantly reduced during the financial crisis, and the trend has not been reverted or even stopped during the post-crisis period. The findings from the post-crisis sub-sample provide new evidence on increased tail dependence in the global real estate market in recent years.

2. Methodologies

Our estimation strategy involves three stages. First, we use an AR(1)-GJR-GARCH(1,1) model (Glosten et al., 1993) to filter the returns to obtain their corresponding residual series. AR-GJR models can capture asymmetric effects on the volatility between two time series, i.e., negative innovations to the returns may generate higher volatility than positive innovations of the same magnitude (Gordon and Canter, 1999, Cotter and Stevenson, 2006; Michayluk et al., 2006).

Second, we estimate the marginal distribution from the residuals obtained in the first step non-parametrically through their empirical cumulative distribution. This procedure is routine to prepare the estimated residual series for the copula estimation in the next step. Specifically, copula models require the inputs to be uniformly distributed within the $[0,1]$ range. To meet this requirement, the marginal distribution of residuals is estimated by using the following formula:

$$F_i(x_i) = \frac{1}{T+1} \sum_{t=1}^T 1_{\{x_{i,t} \leq x_i\}} \quad (1)$$

where $1_{\{x_i \leq x_j\}}$ is an indicator function that takes the value of one if the argument is true and zero otherwise.

Finally, we use the copula method to link the univariate marginal distributions derived from the previous steps. A wide range of copula functions is available, as discussed by Joe (1997) and Nelsen (2007). Among these candidates, Gaussian and t copula functions are the most commonly used, although they are not without shortcomings. The Gaussian copula does not enable tail dependence, while the t copula only considers symmetric tail dependence. Both models are not flexible enough for the purpose of our analysis. Therefore, we adopt the Symmetric Joe-Clayton (SJC) copula (Patton et al., 2006), which is a modification of the Joe-Clayton copula in Joe (1997), as this model can provide both upper and lower tail dependence coefficients to quantify the degree of tail dependence.

For simplicity, we use a bivariate case to illustrate the method adopted in this study. For two uniformly distributed residual series X and Y with a marginal distribution function of $u = F_x(x)$, $v = F_y(y)$, their joint distribution defined by the SJC copula is $F(x, y) = C(u, v)$. If both marginal distributions are continuous, then the copula C_{SJC} is uniquely defined as follows⁴:

$$C_{SJC}(u, v) = F(F_x^{-1}(u), F_y^{-1}(v)) \quad (2)$$

Tail dependence is measured by an upper tail dependence coefficient, τ^U , and a lower tail dependence coefficient, τ^L , as given in Equations (3) and (4).

$$\begin{aligned} \tau^U &= \lim_{\xi \rightarrow 1} P(u > \xi | v > \xi) = \lim_{\xi \rightarrow 1} P(v > \xi | u > \xi) \\ &= \lim_{\xi \rightarrow 1} (1 - 2\xi + C(\xi, \xi)) / (1 - \xi) \end{aligned} \quad (3)$$

$$\tau^L = \lim_{\xi \rightarrow 0} P(u \leq \xi | v \leq \xi) = \lim_{\xi \rightarrow 0} P(v \leq \xi | u \leq \xi) = \lim_{\xi \rightarrow 0} C(\xi, \xi) / \xi \quad (4)$$

where $\tau^U \in [0, 1]$, $\tau^L \in [0, 1]$. When $\tau^U = 0$ ($\tau^L = 0$), the upper (lower) tail dependence is absent.

The tail dependence coefficients in Equations (3) and (4) are constant over time. This could be problematic for studies with a long sampling period. To capture the evolution of the tail dependences, we adopt the time-varying SJC copula

⁴ The specific expression of C_{SJC} is

$$\begin{aligned} C_{SJC}(u, v | \tau^U, \tau^L) &= 0.5 \cdot (C_{JC}(u, v | \tau^U, \tau^L) + C_{JC}(1-u, 1-v | \tau^U, \tau^L) + u + v - 1), \text{ where} \\ C_{JC}(u, v | \tau^U, \tau^L) &= 1 - (1 - \{[1 - (1-u)^k]^{-\tau^U} + [1 - (1-v)^k]^{-\tau^L} - 1\})^{-1/k} \end{aligned}$$

proposed by Patton (2006), in which the tail parameters are defined in Equations (5) and (6). This approach is also used by Zhou and Gao (2012) and Hoesli and Reka (2013).

$$\tau_t^L = \Lambda \left(\omega_L + \beta_L \tau_{t-1}^L + \alpha_L \cdot \frac{1}{10} \sum_{j=1}^{10} |u_{t-j} - v_{t-j}| \right) \quad (5)$$

$$\tau_t^U = \Lambda \left(\omega_U + \beta_U \tau_{t-1}^U + \alpha_U \cdot \frac{1}{10} \sum_{j=1}^{10} |u_{t-j} - v_{t-j}| \right) \quad (6)$$

where $\Lambda(x) \equiv (1 + e^{-x})^{-1}$ is a logistic transformation that constrains the tail dependences to stay in (0,1).

3. Data

The securitized real estate markets under investigation are those in the American, Asia-Pacific, and European regional markets, including the US, HK, Japan, Australia, Singapore, the UK, and France. We collect the daily closing total real estate stock price indices of Standard & Poor (S&P) and stock market indices of these markets from Thomson Reuters DataStream for the period between January 2000 and February 2018. The S&P property database shows the components of the broad universe of investable international real estate stocks and reflect their risk and return characteristics. Our sample covers the most important securitized real estate markets in the world, as indicated by both the market capitalization and the history of the securitized real estate market in each country. As shown in Table 1, all of the real estate markets except for the UK have their first REITs listed at least a decade ago. The total capitalization of the REIT markets exceeds USD 100 billion in all of the countries, thus signifying the importance of the real estate sector in the national economy. On the whole, the sample is a good representation of global securitized real estate markets⁵. The common stock market and public real estate market indices used in this study are also given in Table 1.

Our sample consists of approximately 4660 daily observations of real estate and stock price indices from January 2, 2000 to February 6, 2014. We define the return of the price index in market i at time t as $R_{i,t} = 100 \cdot (\ln(P_{i,t}) - \ln(P_{i,t-1}))$, where $P_{i,d}$ denotes the daily price of the price index.

⁵ The majority of the REITs in these markets primarily invest in domestic markets. For example, among the 24 REITs in the UK, only 9 are internationally focused (i.e., mainly invested in overseas real estate markets). Therefore, each REIT index is a good representation of the price movement of the underlying real estate assets in its corresponding country.

Table 1 Market Capitalization and History of REIT Market

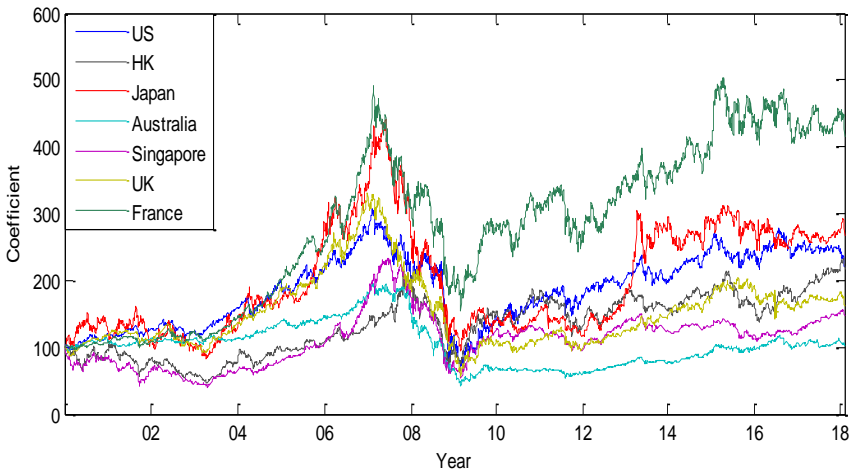
Country/City	Size of REIT market (US \$ million, FTSE)	First REITs Listed	Public Real Estate Market Index	Stock Market Index
US	835888	1960	S&P United States Property Index	S&P 500
HK	41554	2003	S&P HK Property Index	Hang Seng Index
Japan	229318	2000	S&P HK Property Index	Nikkei 225
Australia	104234	1971	S&P Australia Property Index	S&P/ASX 200
Singapore	66793	2002	S&P Singapore Property Index	Straits Times Index
UK	88331	2007	S&P UK Property Index	FTSE 100
France	58450	2003	S&P France Property Index	France CAC 40

Source: Standard & Poor Global Property Database

The summary statistics of the returns for the whole sampling period is presented in Panel A of Table 2. The mean daily returns vary across all seven cities/countries, which range from as low as 0.0001 in Australia to as high as 0.0306 in France. The returns show significant skewness and kurtosis, i.e., the tails are fat and asymmetric in all of the cities/countries. The Jarque–Bera normality test also rejects the null hypothesis that the returns follow a Gaussian distribution. These results lead to the adoption of non-Gaussian models to describe the marginal distributions of the returns and the dependence structures between these cities/countries.

Figure 1 presents the movement of the daily prices of the seven cities/countries. Daily prices in the different cities/countries tend to move in similar directions and fluctuate dramatically during the financial crisis (2007–2009), but the patterns are less consistent after 2009. The dependence structures among the cities/countries might have changed since the Global Financial Crisis. Therefore, in the latter parts of the paper, we investigate this change in dependence by dividing the whole period into three sub-periods: pre-crisis (2000–2006), crisis (2007–2009), and post-crisis (2010–2018). The descriptive statistics of these three sub-samples are given in Panels B to D in Table 2. In general, the crisis period has the lowest returns, even negative returns for most cities/countries, compared with those in the other two periods. Conversely, during the crisis, the standard deviations of the daily returns of the real estate indices of all the cities/countries are the largest, i.e., they are most volatile in the crisis period.

Figure 1 Securitized Real Estate Price Indices (2000–2018)
(Normalized at 100 on 1 January 2000)



Source: Standard & Poor Global Property Database

Table 2 Descriptive Statistics of Daily Returns (%)**Panel A Entire Studied Period (2000–2018)**

	US	HK	Japan	Australia	Singapore	UK	France
Mean	0.0170	0.0168	0.0211	0.0001	0.0081	0.0101	0.0306
Max	17.1015	10.8817	14.0702	7.1326	10.4226	9.1956	7.8700
Min	-21.8432	-10.2967	-12.0222	-10.7370	-9.3128	-15.4914	-8.1239
Std Dev	1.8032	1.5141	1.7439	1.1707	1.3700	1.3765	1.2857
Skewness	-0.2294	-0.0621	0.0202	-0.7663	0.1995	-0.5650	-0.0803
Kurtosis	24.5568	7.6776	7.6253	13.7866	9.5468	13.0731	7.2969
Normality	0.001	0.001	0.001	0.001	0.001	0.001	0.001

Panel B Pre-Crisis Period (2000–2006)

	US	HK	Japan	Australia	Singapore	UK	France
Mean	0.0556	0.0158	0.0682	0.0337	0.0309	0.0659	0.0779
Max	4.5183	6.9378	8.7086	3.3387	10.4226	8.9543	7.8700
Min	-5.1737	-9.4126	-6.0773	-3.5705	-9.3128	-4.8848	-4.4670
Std Dev	0.8660	1.5248	1.7457	0.6613	1.5332	0.8819	0.8311
Skewness	-0.4786	-0.0490	0.2532	-0.1180	0.2088	0.3908	0.1093
Kurtosis	5.9334	5.8658	4.3801	4.8386	7.7398	12.4292	10.1659
Normality	0.001	0.001	0.001	0.001	0.001	0.001	0.001

Panel C Crisis Period (2007–2009)

	US	HK	Japan	Australia	Singapore	UK	France
Mean	-0.0859	0.0168	-0.1179	-0.1273	-0.0436	-0.1419	-0.0493
Max	17.1015	10.8817	14.0702	7.1326	8.7677	9.1956	6.9893
Min	-21.8432	-10.2967	-11.0158	-10.7370	-8.9045	-10.1860	-8.1239
Std Dev	3.7527	2.3220	2.5621	2.2273	2.0909	2.4715	2.0605
Skewness	-0.0460	0.0110	-0.0133	-0.5171	0.2490	-0.0710	0.0063
Kurtosis	7.6279	5.1375	5.5047	5.8299	5.1758	4.1681	4.1356
Normality	0.001	0.001	0.001	0.001	0.001	0.001	0.001

Panel D Post-Crisis Period (2010–2018)

	US	HK	Japan	Australia	Singapore	UK	France
Mean	0.0216	0.0176	0.0315	0.0180	0.0075	0.0179	0.0190
Max	9.4796	6.0769	8.7184	3.9467	2.8274	5.6068	7.1232
Min	-9.5897	-5.5811	-12.0222	-3.4719	-4.3916	-15.4914	-6.1939
Std Dev	1.1931	1.0592	1.3147	0.9216	0.7378	1.1401	1.2364
Skewness	-0.1953	-0.3195	-0.1162	0.0032	-0.5787	-1.5179	-0.0600
Kurtosis	9.7641	6.1292	11.5530	4.1518	5.8995	24.0595	5.5008
Normality	0.001	0.001	0.001	0.001	0.001	0.001	0.001

Note: Means are in percentage. Normality is the p -value of the Jarque–Bera test.

4. Empirical Findings

We first filter the return series of the real estate securities with the AR(1)-GJR-GARCH(1,1) model to obtain the independent and identically distributed (*i.i.d.*) residuals, which are used to construct the marginal distributions for returns in the next stage. The estimated results from the filter are shown in Table 3. The parameter used to describe the asymmetry effect (i.e., γ) is significant at the 1% level for almost all of the cities/countries in all of the sub-periods, thus indicating that the securitized real estate indices are more sensitive to negative news than positive news. Moreover, the estimated γ in the crisis period (i.e., Panel B in Table 3) is much larger than that in the other sub-periods. All of the preliminary evidence suggests that the fatness and asymmetry of tails should be considered in the steps to follow.

4.1 Real-Estate-Only Analysis

In this section, we present the results that are relevant to real-estate-only portfolio management. Specifically, we estimate the tail dependence between the real estate securities market and city/country pairs. With residuals obtained from the previous step, we construct the marginal distributions of the returns by estimating the empirical cumulative distribution. The results are then linked by SJC copula functions to estimate the tail dependence coefficients as defined in Equations (3) and (4). With the seven cities/countries included in our sample, we obtain 21 city/country pairs. In order to verify that the SJC copula is a better estimator than the Gaussian copula, we compare the Akaike information criterion (AIC) statistics between these two models in the last two columns of Table 4. Except for the two tail-independent city/country pairs (i.e., US-UK and HK-Australia), all other city/country pairs have smaller AIC values in their SJC copula models. We therefore use an SJC copula estimator in the rest of the analysis.

Table 4 also gives the tail dependence coefficients estimated from a static SJC copula. These are tail dependence measurements over the entire sampling period. The results suggest that the US and Asia-Pacific real estate markets are tail independent, whilst most of other city/country pairs are both upper- and lower-tail dependent. There are a few exceptions, such as HK-France, Japan-France, Japan-UK and Australia-France which are lower-tail dependent only. The results need to be interpreted with caution, as the static SJC copula does not take into account the dynamics of tail dependence over time. We further explore this issue by first analyzing the data by sub-period, and then estimating the dynamic tail dependence coefficients.

Table 3 Estimation Results from AR(1)-GRACH(1,1) Model**Panel A Pre-Crisis Period (2000–2006)**

	US	HK	Japan	Australia	Singapore	UK	France
c	0.0523 ^b	0.0264	0.0712 ^a	0.0306 ^b	0.0832 ^b	0.0752 ^b	0.0878 ^b
s	0.0926 ^b	0.1207 ^b	0.1193 ^b	--	0.0304	0.0586 ^b	--
ω	0.0424 ^b	0.0159 ^b	0.0129 ^b	0.0262 ^b	0.0213 ^b	0.0127 ^b	0.0662 ^b
α	0.8231 ^b	0.9450 ^b	0.9478 ^b	0.8708 ^b	0.9144 ^b	0.9022 ^b	0.8204 ^b
β	0.0765 ^b	0.0268 ^b	0.0479 ^b	0.0568 ^b	0.0581 ^b	0.0552 ^b	0.0316 ^b
γ	0.0852 ^b	0.0431 ^b	0.0016	0.0251	0.0422 ^b	0.0634 ^b	0.0976 ^b

Panel B Crisis Period (2007–2009)

	US	HK	Japan	Australia	Singapore	UK	France
c	-0.0811	0.0013	-0.1220	-0.0423	-0.0322	-0.1515 ^a	-0.0297
s	-0.1574 ^b	--	0.0608	0.0219	--	--	--
ω	0.0469	0.0560 ^b	0.0943 ^b	0.1035 ^b	0.0222	0.0687 ^a	0.1439 ^b
α	0.8988 ^b	0.8941 ^b	0.9059 ^b	0.8185 ^b	0.8929 ^b	0.9063 ^b	0.8508 ^b
β	0.0476 ^b	0.0518 ^b	0.0308 ^b	0.0857 ^b	0.0602 ^b	0.0704 ^b	0.0798 ^b
γ	0.1072 ^b	0.0897 ^b	0.0974 ^b	0.1682 ^b	0.0936 ^b	0.0278	0.0764 ^a

Panel C Post-Crisis period (2010–2018)

	US	HK	Japan	Australia	Singapore	UK	France
c	0.0231	0.0219	0.0119	0.0206	0.0128	0.0312	-0.0044
s	0.0057	0.0867 ^b	0.0909 ^b	0.0106	0.0828 ^b	0.0141	0.0629 ^b
ω	0.0092 ^b	0.0156 ^b	0.0415 ^b	0.0169 ^b	0.0053 ^b	0.0480 ^b	0.0450 ^b
α	0.8663 ^b	0.9533 ^b	0.8416 ^b	0.9280 ^b	0.9570 ^b	0.8170 ^b	0.8859 ^b
β	0.0943 ^b	0.0098	0.0916 ^b	0.0214 ^b	0.0102	0.1006 ^b	0.0116 ^a
γ	0.0788 ^b	0.0425 ^b	0.1112 ^b	0.0596 ^b	0.0589 ^b	0.1037 ^b	0.1513 ^b

Note: c , s , ω , α , β , and γ are parameters in our AR (1)-GARCH (1,1) model that is specified as $r_{it} = c_i + s_i \cdot r_{it-1} + \varepsilon_{it}$, $\varepsilon_{it} = h_{it} \cdot z_{it}$, $z_{it} \sim N(0,1)$, and $h_{it}^2 = \omega_i + \alpha_i \cdot \varepsilon_{it-1}^2 + \beta_i \cdot h_{it-1} + \gamma \cdot \varepsilon_{it-1}^2 \cdot I_{\{\varepsilon_{it-1} < 0\}}$. We use the Ljung-Box test to check whether a time series is auto-correlated. If the null hypothesis of 'non-autocorrelation' is not rejected, the autocorrelation component will be dropped from the AR (1)-GARCH (1,1) model, and the value of s will be marked with '--' in the table. ^a denotes significance at the 5% level, ^b denotes significance at the 1% level

Table 5 reports the tail dependence coefficient estimates by sub-period. Several conclusions can be drawn from Table 5. First, the number of city/country pairs that exhibit tail dependence increases significantly at the 5% level during the Global Financial Crisis (13 and 16 pairs show upper tail and lower tail dependence, respectively; see last row in Table 5) compared with the number of city/country pairs before 2007 (only two pairs with upper tail dependence

and six pairs with lower tail dependence). This pattern remains largely unchanged during the post-crisis period. The number of city/country pairs with lower tail dependence even increases to 18. Therefore, we conclude that the financial turmoil exerts a significant and long-lasting effect on the dependence structures among cities/countries. The international real estate securities market used to be a good diversification vehicle, as suggested by the low tail dependence coefficients for the period of 2000 to 2006. However, these diversification benefits have decreased notably since 2007. Surprisingly, a significant increase in city/country pairs with lower tail dependence is observed. As diversification matters the most during market downturns, our findings suggest that most international securitized real estate markets cannot offer the same level of protection now as they did in the pre-crisis period.

Table 4 Comparison of Estimated Results from SJC and Gaussian Copulas

		Upper tail coefficient	Lower tail coefficient	AIC(SJC)	AIC(Gaussian)
US	HK	0.0001	0.0206	-2.6514	-2.3113
	Japan	0.0001	0.0008	-0.1787	-0.0145
	Australia	0.0001	0.0099	-1.6652	-1.3557
	Singapore	0.0008	0.0498	-3.8976	-3.5771
	UK	0.0856 ^b	0.1496 ^b	-6.7408	-6.8728
	France	0.0766 ^b	0.1431 ^b	-6.9430	-6.6297
HK	Japan	0.0732 ^b	0.2311 ^b	-7.4980	-7.4883
	Australia	0.0726 ^b	0.1548 ^b	-6.7090	-6.9491
	Singapore	0.2724 ^b	0.4184 ^b	-9.7094	-9.6159
	UK	0.0403 ^b	0.1506 ^b	-6.3528	-6.2854
	France	0.0203	0.1380 ^b	-6.1226	-6.0383
Japan	Australia	0.0500 ^b	0.1440 ^b	-6.6750	-6.3928
	Singapore	0.0457 ^b	0.2429 ^b	-7.5510	-7.4360
	UK	0.0242	0.0835 ^b	-5.3470	-5.3226
	France	0.0073	0.0913 ^b	-5.1814	-4.8989
Australia	Singapore	0.0677 ^b	0.1791 ^b	-7.1290	-6.8824
	UK	0.0287 ^a	0.0554 ^b	-4.9806	-4.9007
	France	0.0181	0.0636 ^b	-4.8935	-4.7365
Singapore	UK	0.0328 ^a	0.1779 ^b	-6.6004	-6.5920
	France	0.0316 ^a	0.1561 ^b	-6.4760	-6.3674
UK	France	0.3370 ^b	0.4768 ^b	-10.3347	-10.0351

Note: ^a denotes significance at the 5% level, ^b denotes significance at the 1% level.

Table 5 Tail Dependence Estimation by Sub-Period

		Pre-crisis (2000–2006)		Crisis (2007–2009)		Post-crisis (2010–2018)		
		upper tail	lower tail	upper tail	lower tail	upper tail	lower tail	
US	HK	0.0001	0.0053	0.0279	0.0093	0.0002	0.0469	
	Japan	0.0001	0.0001	0.0001	0.0196	0.0005	0.0092	
	Australia	0.0001	0.0001	0.0026	0.0022	0.0001	0.0586	
	Singapore	0.0001	0.0111	0.0320	0.0775	0.0050	0.0772 ^b	
	UK	0.0360	0.0166	0.1069 ^a	0.2816 ^b	0.1486 ^b	0.2120 ^b	
	France	0.0057	0.0003	0.1070 ^a	0.2727 ^b	0.1820 ^b	0.2319 ^b	
HK	Japan	0.0189	0.1582 ^b	0.2367 ^b	0.3736 ^b	0.0948 ^b	0.2398 ^b	
	Australia	0.0233	0.0271	0.1617 ^b	0.3511 ^b	0.0951 ^b	0.2063 ^b	
	Singapore	0.2265 ^b	0.3177 ^b	0.3878 ^b	0.5688 ^b	0.2650 ^b	0.4531 ^b	
	UK	0.0116	0.1260 ^b	0.0896	0.1407 ^b	0.0598 ^a	0.1957 ^b	
	France	0.0001	0.1050	0.0585	0.1935 ^b	0.0669 ^a	0.1461 ^b	
	Japan	Australia	0.0019	0.0364	0.1953 ^b	0.3344 ^b	0.0708 ^a	0.1825 ^b
Japan	Singapore	0.0001	0.1838	0.2809 ^b	0.3606 ^b	0.0460 ^a	0.2600 ^b	
	UK	0.0014	0.0479	0.0360	0.1508 ^b	0.0660 ^a	0.1001 ^b	
	France	0.0001	0.0590	0.0169	0.1960 ^b	0.0378	0.0760 ^b	
	Australia	Singapore	0.0226	0.0581 ^b	0.1547 ^b	0.2832 ^b	0.1012 ^b	0.2370 ^b
	UK	0.0244	0.0001	0.1182 ^b	0.0674	0.0186	0.1370 ^b	
	France	0.0068	0.0006	0.0933 ^b	0.0867 ^a	0.0152	0.1119 ^b	
Singapore	UK	0.0028	0.1249 ^b	0.1158 ^a	0.2127 ^b	0.0480	0.2292 ^b	
	France	0.0001	0.0787	0.1087 ^a	0.2805 ^b	0.0756 ^b	0.1804 ^b	
UK	France	0.0849 ^b	0.2437 ^b	0.5218 ^b	0.6049 ^b	0.4911 ^b	0.5755 ^b	
Number of significant pairs		2	6	13	16	13	18	

Note: ^a denotes significance at the 5% level, ^b denotes significance at the 1% level

However, the picture is not completely gloomy. Specifically, not all of the cities/countries are equally affected by the financial crisis. For example, tail dependences between the US and other cities/countries are insignificant even during the turmoil, consistent with the results in Zhou and Gao (2012). In addition, the tail dependence among European countries (as high as 0.5218 and 0.6049 for the upper and lower tail dependence, respectively, during the crisis) is much stronger than that among the Asian cities/countries. We also observe a closer relationship among cities/countries in the same continent than the city/country pairs in different continents. For example, the coefficients of the UK–France pair in all three periods are much larger than the corresponding coefficients of the UK–HK pair. However, identifying the causes of these differences is beyond the scope of this study, as the focus of this study is to investigate whether and how tail dependence among cities/countries varies over time and across geographic regions. Our findings strongly support the notion that the interdependence among international securitized real estate markets is complex and dynamic. We conclude that the landscape of international securitized real estate markets in terms of tail dependence has changed

fundamentally since the Global Financial Crisis. Markets are much more dependent on each other, especially during difficult times. Although the global economy has been gradually recovering from the crisis, strong tail dependence (lower tail dependence in particular) still persists. Investors and fund managers should take this into account when considering international securitized real estate products in their portfolios.

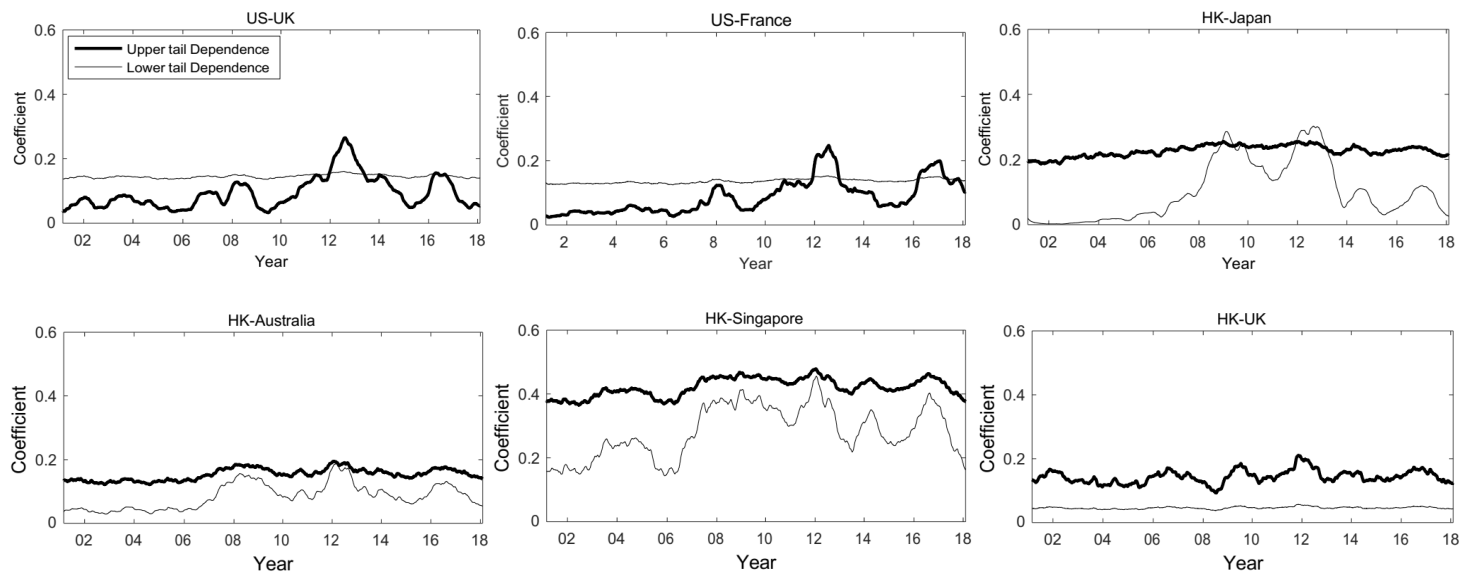
The findings in Table 5 are interesting and informative. However, the definition of the sub-periods is subjective, and might introduce errors in the analysis. For example, one may wonder if it is necessary to split the 2007 to 2018 period into crisis and post-crisis sub-periods. To answer this question, we use a time-varying copula estimator as defined in Equations (5) and (6) to re-estimate the tail dependence coefficients for all city/country pairs with at least one significant tail.

As the data are daily series, the original coefficient estimate series are quite noisy. In order to show the trend more clearly, we smooth the estimates by using a 250-day rolling window⁶. The time-varying lower and upper tail dependence for each country-pair are presented in Figure 2. The patterns identified in this figure are very similar to those in Table 5. For example, tail dependence has increased on the whole; within-region tail dependence is stronger than cross-region dependence; and HK-Singapore and UK-France have the strongest tail dependence throughout the whole sampling period.

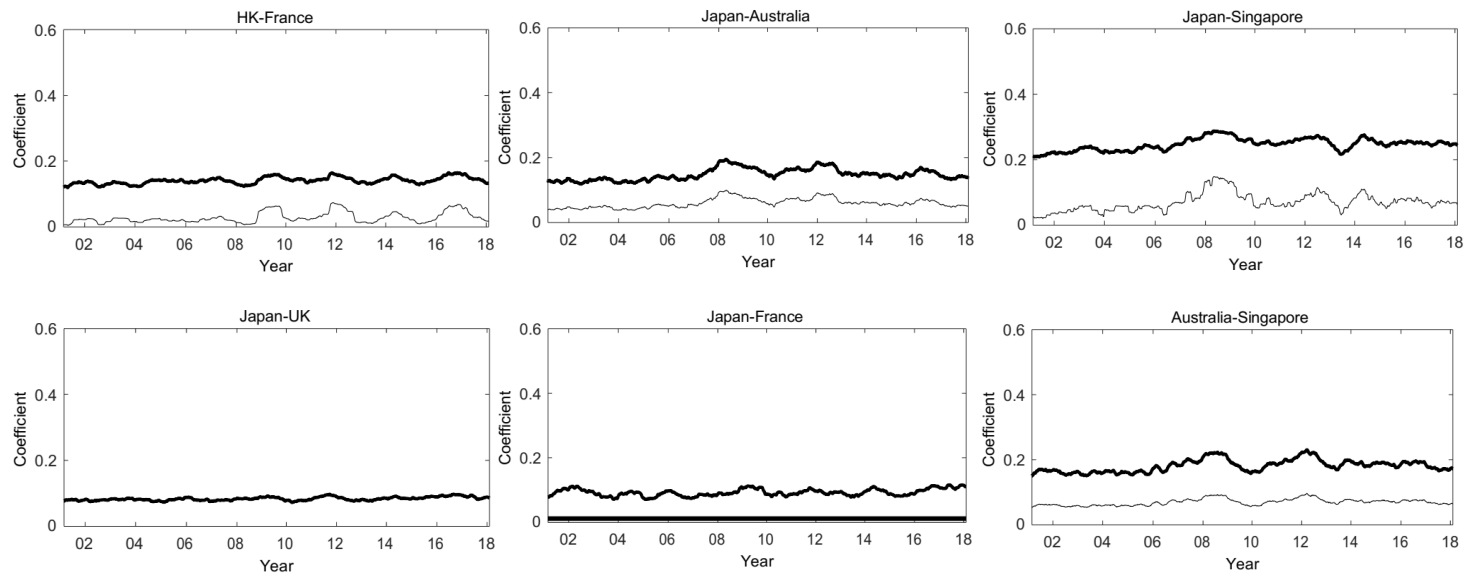
More importantly, Figure 2 shows two peaks in the lower-tail dependence between 2008-2009 and 2011-2013 for most city/country pairs. These could be attributed to the Global Financial Crisis and the European Debt Crisis, with an approximately one-year lag time. In addition, we can also find a third peak around 2016 and 2017, which coincides with Brexit. This is consistent with existing findings that negative shocks have significant impacts on the linkages between different real estate markets. It is worth noting that the pattern is not restricted to EU countries. For example, the Australian-Singapore and HK-Japan pairs also demonstrated this bi-modal pattern: the level of lower-tail dependence increased at almost the same magnitude between these two pairs in 2012. Although there are no global events that are equivalent to the Global Financial Crisis around 2012 to explain for the peak, this pattern does indicate that global real estate markets have become more interdependent since the Global Financial Crisis. The findings justify our strategy to analyze tail dependence by using sub-periods. We continue to use this approach in the rest of the analysis because it provides similar results as the dynamic Copula approach, but with more intuitive and economically meaningful interpretations.

⁶ We choose 250 as the window size of smoothing so that each point represents the tail dependence in one year. We have also tried other window sizes, but found that they made little difference.

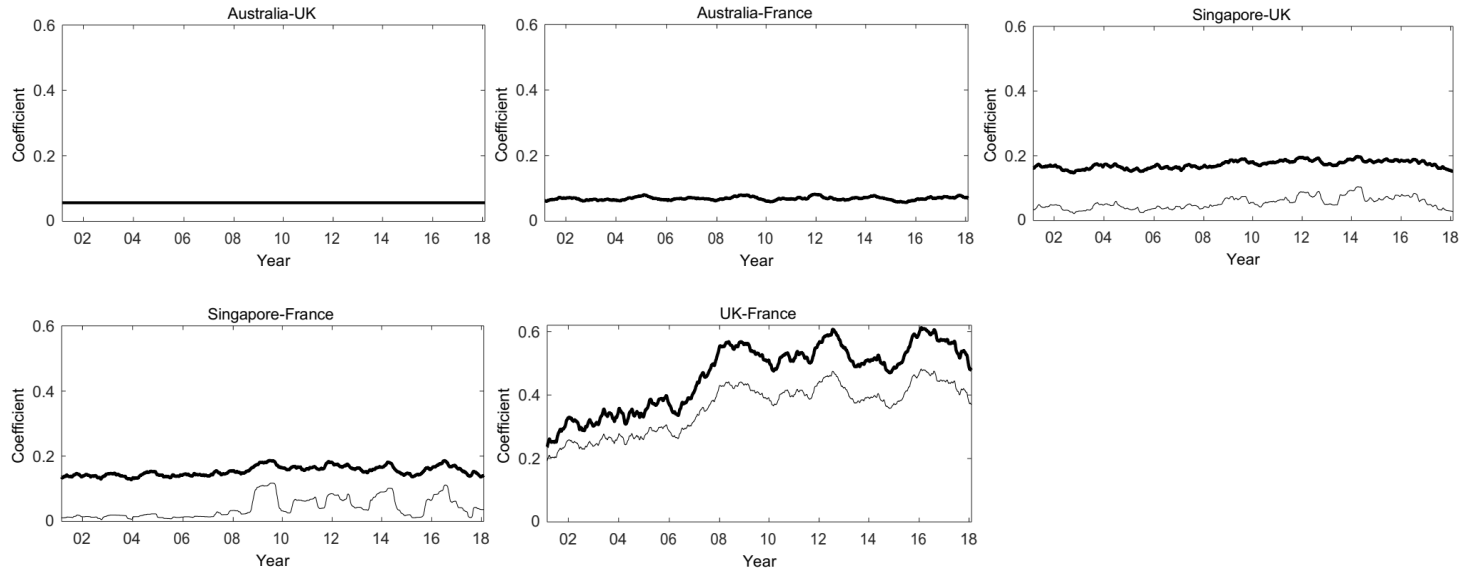
Figure 2 Time-Varying Tail Dependences in Global Real Estate Markets



(Continued...)

(Figure 2 Continued)*(Continued...)*

(Figure 2 Continued)



4.2 Mixed Assets Analysis⁷

Our findings in Table 5 shed some light on the investment strategies for real-estate-only portfolios. The benefits of diversifying into the international real estate securities markets have diminished significantly since 2007. Is the same conclusion also true for mixed-asset portfolios? To answer this question, we analyze the tail dependence between stock markets and securitized real estate markets both at the domestic and international levels.

For each of the seven cities/countries, we first estimate the tail dependence coefficients between its own stock market and the real estate securities market. The results are given in Table 6. Unsurprisingly, stocks and publicly traded real estate securities are significantly related throughout the period for all cities/countries. Both upper tail and lower tail coefficients have increased since the financial crisis in all of these cities/countries except for the upper tail dependence of HK. The conclusion is that the returns of the two asset classes are highly correlated within a country, especially during and after the financial crisis.

Table 6 Dependence between Stock and Public Real Estate Markets at National Level

	Pre-crisis		Crisis		Post-crisis	
	upper tail	lower tail	upper tail	lower tail	upper tail	lower tail
US	0.3784 ^b	0.3116 ^b	0.5785 ^b	0.6797 ^b	0.4389 ^b	0.4593 ^b
HK	0.6871 ^b	0.7095 ^b	0.6389 ^b	0.7676 ^b	0.5330 ^b	0.6721 ^b
Japan	0.3924 ^b	0.5117 ^b	0.4797 ^b	0.6683 ^b	0.4845 ^b	0.6132 ^b
Australia	0.2810 ^b	0.3180 ^b	0.4043 ^b	0.5555 ^b	0.4088 ^b	0.4983 ^b
Singapore	0.4706 ^b	0.5306 ^b	0.6560 ^b	0.7638 ^b	0.5133 ^b	0.6678 ^b
UK	0.1941 ^b	0.3929 ^b	0.4054 ^b	0.5481 ^b	0.4320 ^b	0.4860 ^b
France	0.0237	0.1611 ^b	0.3601 ^b	0.5763 ^b	0.4920 ^b	0.5003 ^b
Number of significant pairs	6	7	7	7	7	7

Note: ^b denotes significance at the 5% level.

This picture changes when we investigate the tail dependence between the returns of stocks in each city/country and the returns of real estate securities in other cities/countries. We estimate the upper and lower tail dependence coefficients for 42 city/country pairs formed among the seven cities/countries under investigation. The results are presented in Table 7.

⁷ The dynamic SJC copula estimator shows similar patterns as identified in Tables 6 and 7. Therefore, the results are not presented here, but available from the authors upon request.

Table 7 Dependence between Stock and Public Real Estate Markets at International Level

Stock	RE	Pre-crisis		Crisis		Post crisis	
		upper tail	lower tail	upper tail	lower tail	upper tail	lower tail
US	HK	0.0001	0.0505	0.0954 ^b	0.0214	0.0201	0.0908 ^b
	Japan	0.0001	0.0001	0.0001	0.0505	0.0110	0.0251
	Australia	0.0001	0.0001	0.0065	0.0012	0.0001	0.0432
	Singapore	0.0001	0.0217	0.0901 ^a	0.0964 ^a	0.0363	0.1244 ^b
	UK	0.0700 ^b	0.0989 ^b	0.1356 ^b	0.3287 ^b	0.2366 ^b	0.2527 ^b
	France	0.0045	0.0180	0.1392 ^b	0.3756 ^b	0.3019 ^b	0.2839 ^b
HK	US	0.0001	0.0032	0.0546	0.0067	0.0001	0.0417
	Japan	0.0505	0.1705 ^b	0.2089 ^b	0.3861 ^b	0.1131 ^b	0.2682 ^b
	Australia	0.0554 ^a	0.0353	0.1614 ^b	0.3216 ^b	0.0894 ^b	0.2132 ^b
	Singapore	0.2588 ^b	0.3265 ^b	0.5137 ^b	0.5301 ^b	0.2421 ^b	0.4478 ^b
	UK	0.0241	0.1196 ^b	0.1318 ^b	0.1279 ^b	0.0565	0.2095 ^b
	France	0.0001	0.1251	0.1316 ^a	0.1754 ^b	0.0336	0.1866 ^b
Japan	US	0.0001	0.0001	0.0006	0.0192	0.0012	0.0012
	HK	0.1358 ^b	0.2860 ^b	0.2347 ^b	0.5127 ^b	0.1174 ^b	0.2375 ^b
	Australia	0.0203	0.0691 ^a	0.1917 ^b	0.3715 ^b	0.0252	0.2368 ^b
	Singapore	0.0236	0.2912 ^b	0.2549 ^b	0.4805 ^b	0.0787 ^a	0.2846 ^b
	UK	0.0169	0.0602 ^a	0.0337	0.1539 ^b	0.0375	0.1178 ^b
	France	0.0001	0.0753	0.0373	0.1801 ^b	0.0106	0.1032 ^b
Australia	US	0.0001	0.0001	0.0551	0.0027	0.0003	0.0184
	HK	0.1278 ^b	0.2505 ^b	0.3781 ^b	0.5111 ^b	0.1608 ^b	0.3339 ^b
	Japan	0.0393	0.1675 ^b	0.2534 ^b	0.4421 ^b	0.1546 ^b	0.2791 ^b
	Singapore	0.0725 ^b	0.2333 ^b	0.3600 ^b	0.4265 ^b	0.1276 ^b	0.3631 ^b
	UK	0.0186	0.0480	0.0974 ^a	0.1257 ^b	0.0365	0.1713 ^b
	France	0.0004	0.0610	0.0337	0.1741 ^a	0.0174	0.1526 ^b
Singapore	US	0.0001	0.0031	0.0340	0.0724	0.0028	0.0500 ^a
	HK	0.2287 ^b	0.4011 ^b	0.4283 ^b	0.5889 ^b	0.2852 ^b	0.4567 ^b
	Japan	0.0200	0.2064 ^b	0.2379 ^b	0.3618 ^b	0.0828 ^b	0.2672 ^b
	Australia	0.0287	0.0744 ^b	0.1246 ^a	0.2626 ^b	0.0643 ^a	0.2423 ^b
	UK	0.0058	0.1583 ^b	0.1667 ^b	0.2185 ^b	0.0630 ^a	0.2045 ^b
	France	0.0001	0.1262	0.1606 ^b	0.2962 ^b	0.0581 ^a	0.1893 ^b
UK	US	0.1255 ^b	0.0364	0.2143 ^b	0.2742 ^b	0.1523 ^b	0.1833 ^b
	HK	0.0401	0.1738 ^b	0.1668 ^b	0.2205 ^b	0.0744 ^a	0.2122 ^b
	Japan	0.0001	0.0839	0.0262	0.1898 ^b	0.0328	0.0949 ^b
	Australia	0.0252	0.0077	0.0663	0.0586	0.0184	0.0860 ^b
	Singapore	0.0298	0.1494 ^b	0.2016 ^b	0.2723 ^b	0.0717 ^a	0.2494 ^b
	France	0.0210	0.1380 ^b	0.3635 ^b	0.5495 ^b	0.4009 ^b	0.4348 ^b
France	US	0.1126 ^b	0.0624 ^a	0.2261 ^b	0.3255 ^b	0.1522 ^b	0.2032 ^b
	HK	0.0631 ^a	0.1502 ^b	0.1931 ^b	0.1906 ^b	0.0665 ^a	0.1948 ^b
	Japan	0.0001	0.0895	0.0173	0.2102 ^b	0.0456	0.1019 ^b
	Australia	0.0029	0.0327	0.0638	0.0693	0.0063	0.1014 ^b
	Singapore	0.0469	0.1520 ^b	0.2017 ^b	0.2737 ^b	0.0659 ^a	0.2490 ^b
	UK	0.1235 ^b	0.3250 ^b	0.3405 ^b	0.5386 ^b	0.3866 ^b	0.4157 ^b
Number of significant pairs		11	22	29	33	24	37

Note: ^a denotes significance at the 5% level, ^b denotes significance at the 1% level.

Similar to the pattern identified in Tables 5 and 6, the linkage between domestic stock market and international real estate also increased after the start of the financial crisis. This result can be justified by the increase in the number of city/country pairs that exhibit tail dependence. However, two aspects deserve further discussion. First, the level of tail dependence in Table 7, as measured by the absolute values of the tail dependence coefficients, is much smaller than that in Table 6. This finding indicates that investing in the international real estate securities markets still offers significant diversification benefits compared to investing in the domestic real estate securities market. Second, although the interdependence between domestic stock market and international real estate securities markets increased during the financial crisis, the magnitude of the changes is less than that reported in Table 5. For example, the number of city/country pairs with significant upper tail dependence actually drops from 29 in the crisis period to 24 in the post-crisis period (see Table 6), while the same statistics are maintained in Table 5. The increase in city/country pairs with lower tail dependence is also much lower after 2007 in Table 6 as opposed to Table 5.

In conclusion, diversification benefits can still be gained by investing across asset classes and geographic regions, although the advantages have significantly reduced during and after the financial crisis. Generally, investors are recommended to form mixed-asset portfolios that consist of both stocks and real estate securities from different geographic regions. Real-estate-only portfolios, regardless of their level of geographic distribution, cannot offer enough diversification benefits as they did before the Global Financial Crisis. This finding is particularly true when markets are under the influence of negative shocks.

5. Conclusion

We study the dependence structures in seven major public real estate markets (i.e., the US, HK, Japan, Australia, Singapore, the UK, and France). A flexible form of the copula model, i.e., the SJC copula, is adopted to quantify the tail dependence in the return series. In contrast to previous studies that have evaluated only long-run correlation or the dependence of real estate markets, we model the changes of the dependence between city/country pairs by using three subsamples that cover the pre-crisis, crisis, and post-crisis periods. The empirical results confirm that a large number of city/country pairs have changed from tail independence to tail dependence since the crisis. The benefits of diversifying into international real estate securities markets have significantly decreased, especially for real-estate-only portfolios. Our empirical investigation is an extension of Zhou and Gao (2012) and Hoesli and Reka (2013) by emphasizing international linkages and using sub-periods to investigate whether dependence changes over time. The findings from the post-

crisis sub-sample provide new evidence on increased tail dependence in the global real estate market in recent years.

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